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Problem Chosen

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2012 Mathematical Contest in Modeling (MCM) Summar Sheet
(Attach a cop of this page to each cop of our solution paper.)

T pe a summar of our results on this page. Do not include
the name of our school, advisor, or team members on this page.

In this paper, we study the method to arrange the trips of variable days and optimize the occupancy of the campsites. We establish different models based on the constraint conditions. Before calculation, several assumptions are made for simplicity:

- Customers can not change the propulsion or duration during the trip.
- Both of the oar-powered rubber raft and the motorized boat can travel at most 8 hours per day.
- Boats can not stop during the voyage unless reaching the terminal of the day, so each day two boats can only meet at most one time.

Other assumptions please see page 3.

With these assumptions, we build our Model 1 which is simple but sufficient to show our fundamental idea. Then to optimize the result of Model 1, we reduce some constraint conditions, and get Model 2. Finally, to improve Model 2, we get the Model 3, which is more complex and more familiar with our real traveling experience.

In Model 1, we arrange only one route for each kind of trip and the tourists can only travel a constant distance everyday. In Model 2, we allow the tourists to change their traveling distance. In Model 3, we allow that each kind of trips has several routes and in each route tourists can travel different distances everyday.

We use greedy algorithm and genetic algorithm to simulate all the models, and get the approximate optimal solutions by comparing the number of the trips, the occupancy of the campsite in average and the contacts per day during a trip. We also discuss the strengths and weaknesses of each model in detail. Furthermore, we do the sensitivity analysis to Y and I (which means the interval of different starting day) and find out the effects of the two variables to the optimal result.

In order to improve the feeling of the tourists, we establish the “free way” model. The traveling company can set the number of oar-powered rubber rafts and motorized boats everyday according to their studies of the market.

We aim at the highest number of trips in the premise of an acceptable number of contacts. However, different people emphasize different point, so the results will be quite different.

MEMO

February 14,2012

TO: RIVER MANAGER
FROM: TEAM 14125
SUBJECT: SCHEDULE OF WHITE WATER RAFTING

We make a schedule of the rafting trips. Here are some attentions:

- Customers can not change the propulsion and duration during the trip.

- The speed of oar-powered rubber raft and motorized boat can not be changed.

- Both of the oar-powered rubber raft and the motorized boat can travel at most 8 hours per day.
- Customers go white-water rafting only in daytimes and they have to go to campsites for rest in night.
- No two sets of campers can occupy the same site at the same time.

There are 3 ways to arrange the schedule:

- Each kind of trips provides only one route and tourists can only travel a constant distance everyday.
- Each kind of trips provides only one route, but tourists travel different distances everyday.
- Each kind of trips provides several routes and in each route, tourists can travel different distances everyday.

The aim of the schedule:

- Utilize the campsites in the best way.
- The maximum of the trips.
- Minimal contact with other groups of boats on the river.

“Free way” arrangement can improve the quality of the trips and allow the tourists to have their own independent travels.

DESIGNING THE OPTIMAL SCHEDULE OF THE WHITE WATER RAFTING

Mathematical Contest in Modeling

Group #14125

2/14/2012

Contents

1. Introduction

White water rafting is an adventure sport in which a raft is used to navigate a river or any water body. It is usually done on rapids or white water to keep up the excitement. The sport has been popular since the mid 80's

Fig.1 White water rafting

White water rafting suits all age groups as various types of rapids are available for different tastes and experience. Almost anyone with a reasonable health can go for white water rafting. It is a good way to spend a vacation, travel abroad, be among the nature, get the sun and of course get a tan. It is considered a good way to release the stress. It is considered a good way to release the stress. The expeditions range from a day to a month. White water rafting is now widely practiced around the world.

There are three main tasks in this paper as follows:

- Develop the best schedule with an optimal mix of trips, of varying duration and the way of propulsion that will utilize the campsites in the best way.
- Determine the carrying capacity of the river that means the maximum of X.
- Allow trips to enjoy a wilderness experience, with minimal contact with other groups of boats on the river.

The problem shows the connection with the schedule and the carrying capacity of the river, which leads to determine the total number of trips and the feeling of the wilderness experience. With the reasonable schedule the customers will enjoy the wilderness experience and relax themselves. Besides, with the growth of the trips, the profit will increase. We will analyze the problem and find a way to optimize the schedule and utilize the campsite in the best way which contributes to a more comfortable journey.

We analyze the problem and formulate the goal in order to allow the highest number of trips to travel down the river, get the highest campsite occupancy, and make the trips less contact with other boats. We can find that the contact increases with the growth of the number of trips by simulation. By restricting the number of contact we translate it into a constraint condition and translate the bi-objective programming into single-objective programming.

2. Conditions, Hypotheses and Symbols

2.1 Conditions:

- The Big Long River is 225 miles long, suppose L =225 miles.
- The trips range from 6 to 18 nights of camping on the river.
- Oar- powered rubber rafts travel on average 4 mph and motorized boats travel on average 8 mph. Suppose $v_1 = 4mph,\ \ v_2 = 8mph$.
- The Big Long River is open only six months in a year
- No two sets of campers can occupy the same site at the same time

2.2 Hypotheses

- Customers can not change the propulsion and duration during the trip.
- The speed of oar-powered rubber raft and motorized boat can not be changed.
- Boats and rafts keep going down during the journey and can not return.
- The views surrounding different campsites are almost the same.
- Both of the oar-powered rubber raft and the motorized boat can travel at most 8 hours per day.
- All trips will not be influenced by the quality of boat and the weather.
- Customers go white-water rafting only in daytimes. They have to go to campsites for rest at night and stay only one night at the same campsites.
- The trips with motorized boats range from 6 to 12 nights of camping on the river and the trips with oar- powered rubber rafts range from 13 to 18 nights of camping on the river.
- Customers have the same interest in every trip ranging from 6-18 nights that means they choose the trips with the same probability.
- The campsite is only for rest in the night. Which campsite you choose makes no difference.
- Boats can not stop during the voyage unless reaching the terminal of the day, so each day two boats can only meet at most one time.
- Considering the environment carrying capacity, the more campsites we use,

the more bad the environment will be. So we suppose the interval of each campsite is at least four miles, then we can get the highest number of campsites is $\frac{225}{4}-1 \approx 55$. So Y is between 18 and 55.

All hypotheses above are designed for simplifying the calculation. Qualitative analysis can be made based on these assumptions.

2.3 Symbols used in the paper

symbols	Explanations
L	the total length of the Big Long River
V_1	the average speed of Oar- powered rubber rafts
V_2	the average speed of motorized boats
T_i	a kind of trip with i nights, e.g. T_6 means a trip with 6 nights camping on the river $(6 \le i \le 18)$
C_i	the ith campsite on the Big Long River. $(1 \le i \le Y)$
Y	the total number of campsites
N_i	the number of campsites that trip T_i come across everyday
$Z_n = \{Z_{n1}, Z_{n2}, ..., Z_{ni}, ...\}$	the set of different kinds of trips in the n^{th} day
I	the interval of different starting day
D_A (or D_B)	the day when trip A (or B) sets out
t_A, t_B	the total days of trip A (or B)

3. Models

3.1 Definitions

Campsite occupancy: Defined as the average ratio of the number of occupied campsite to the total number of the campsite during six months.

Contact: A contact exists when each two trips meet on the journey.

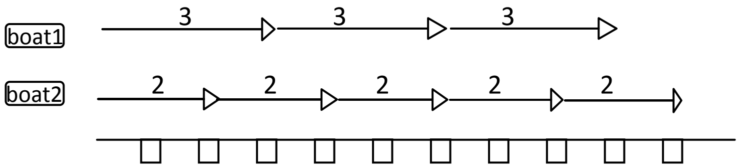
Conflict: A conflict exists when each two trips reach the same campsite at the same time.

Suppose the interval of different starting day is constant in model 1 to 3.

3.2 Model 1

In the first model, each kind of trips provides only one route and tourists can only travel a constant distance everyday.

Figure 2 shows a simplified outline of the first model.



Campsites distributed fairly uniformly throughout the river corridor

Fig2

3.2.1 How it works

- **In what condition can a contact occur:**

If trip A sets out earlier than trip B and trip A arrives at the terminal point later than trip B, trip A and B will meet once during the voyage. In other words, suppose D_A (or D_B) represents the day when trip A (or B) sets out, t_A (or t_B) represents the number of days of trip A (or B).

If $D_A < D_B, D_A + t_A > D_B + t_B$ or $D_A > D_B, D_A + t_A < D_B + t_B$,

then trip A and trip B will meet once during the journey.

- **Choose the best way for the number of contacts to measure the quality of a trip**

The problem demands that every trip should be with minimal contact with other groups of boats on the river so as to enjoy a wilderness experience. What is the best way to measure the quality of a trip in consideration of the number of contacts. We list five ways as follows:

1. The total number of contacts in the rafting season(6 months)
2. The average number of contact each day.
3. The total number of contacts of one trip.
4. The ratio of the total time during contacts to the total time(6 months)
5. The ratio of the number of contacts during a trip to the days of the trip

Finally we choose the 5th way compared with the others. The analyses are as follows:

The first and the second methods are almost the same. They both have a shortcoming that a huge difference in the number of contacts might exist in different days. For example, during a trip, there are 10 contacts in a day, but there are none in another day. Although the average number might be low, the real quality of the trip is not very well. So these two methods can not measure the quality of the trip well; the third method can not work well because different kinds of trips have different number of contacts. For example, one condition is that the number of contacts of a 6-night trip is 18 and of an 18-night trip is 6, another condition is that the number of contacts of a 6-night trip is 6 and of an 18-night trip is 18. The two conditions have the same result, but the reality is that the second condition is better; the fourth method can not operate well because the time of each contact is hard to calculate and huge errors might exist; the fifth method is an upgrade of the third method. After getting the number of contacts and the days of each trip, we take the ratio of the number of contacts during a trip to the days of the trip. Then compare all the ratios. If we can formulate a way to set a limit to the highest ratio, we can guarantee the quality of the trips.

- **Find the feasible route**

Step1. Suppose the number of campsites that the kind of trip T_i come across everyday is N_i:

$$N_i = floor(\frac{Y}{i}) \quad \left(6 \le i \le 18\right)$$

$floor(x)$: means the largest integer not greater than x

Step2 Suppose the set of different kinds of trips that set out in the n^{th} day is $Z_n = \left\{Z_{n1}, Z_{n2}, \ldots Z_{ni}, \ldots\right\}$, $\left(6 \le i \le 18\right)$,(If $\left(i+1\right)\times N_i \le Y$, then get rid of Z_{ni})

Suppose d is the interval of different starting day.

$\forall m \in \left\{1, 2, \ldots, n-1\right\}$ If $\left(n-m\right)\times d < Z_{mi}$ $\left(\forall Z_{mi} \in Z_m\right)$, then

$\forall t \in \left\{1, 2, \ldots, Z_{mi} - (n-m)\times d\right\}$, If $\forall Z_{nj} \in Z_n$ s.t. $\left(t+\left(n-m\right)\times d\right)\times N_{Z_{mi}} \ne t \times N_{Z_{nj}}$,

then we can get a set Z_n.

step3. If Z_n is not empty, then we get a practical arrangement of trips.

3.2.2 Consequences

Taking Y=30 for example, we will get an optimal arrangement of trips during 6 months. There is only one chart here and others are in the appendix:

In order to get the highest occupancy, we suppose the trips set out as many as possible.

T_i	T_6	T_7	T_{10}	T_{15}
The set-out date	Day 1	Day 1	Day 1	Everyday

If I=1, we can get the number of trips is 169, the occupancy is 46.54% and contact is 0.

If I=2, we can get the number of trips is 86, the occupancy is 23.48% and contact is 0

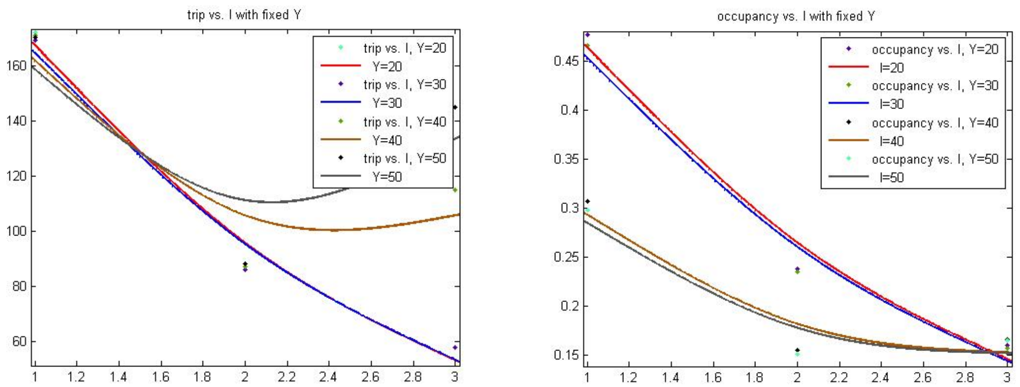
……

We can find that in this way the number of contacts is *zero.*

3.2.3 Sensitivity

We do the sensitivity analysis to Y and I and find the influence of them to the optimal solution.

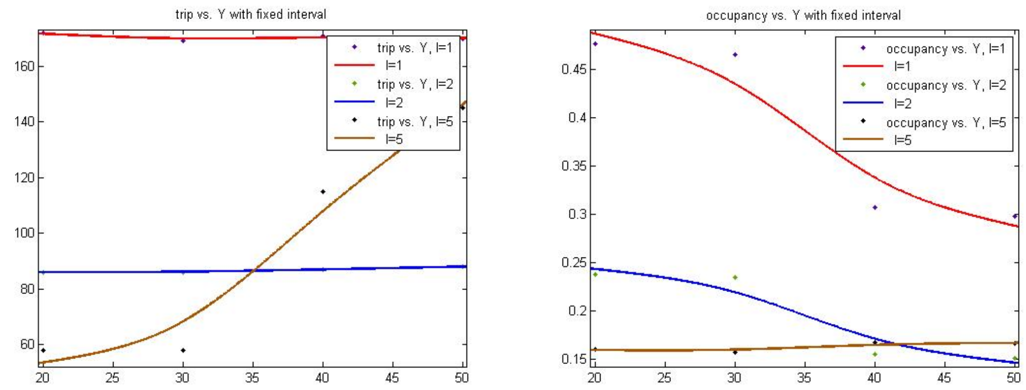
1. To the fixed Y, we analyze I and use the method of smooth spline to get the fitting curve.



(1) (2)

Fig.3

2. To the fixed I, we analyze Y and use the method of smooth spline to get the fitting curve.



(3) (4)

Fig.4

From the figure, we can find when Y increases, the curve of Fig4 (4) tends to be smooth.

3.2.4 Problems in Model 1 and Improvements

1. According to the consequences, we find that when getting the highest occupancy, the number of trips may not reach the highest, because arranging the long-journey boats as many as possible can make the occupancy higher, but also will decrease the number of trips.

When the kinds of trips have been arranged, we can get the “conflict-table”. Still taking Y=30 and interval=2 for example, suppose the set of different kinds of trips is [6,7,10,15]. We can get the “conflict-table” as follows:

The days after setting out / T_i	1×interval	2×interval	3×interval
T_6	/	/	/
T_7	/	/	/
T_{10}	6,7	6	/
T_{15}	7,10	7,10	6,7

Table.1

For example, the number in the fourth row and the second column is (7, 10), which means T_{15} sets out today and in the next starting day T_7, T_{10} can not set out.

Although arranging T_{15} can improve the occupancy, after T_{15} sets out, the kinds of trips that can be chosen are limited and the total number of trips decreases. So when taking occupancy as high as possible and the trips set out as many as possible, then every interval T_{15} will set out; it will leads to the decrease of the number of trips and go against to the diversity of the journey.

In order to solve the problem, we suppose that the same kind of trip T_i can not set out continuously, then in the same example(Y=30, interval=2), the "conflict-table" is as follows:

The days after setting out / T_i	1×interval	2×interval	3×interval
T_6	6	/	/
T_7	7	/	/
T_{10}	6,7,10	6	/
T_{15}	7,10,15	7,10	6,7

Table.2

The result is:

T_i	6	7	10	15
The date of boats set out	Day 4n+1,n=0,1,…,43	Day 1	/	Day 4n+3,n=0,1,…,40

Table.3

Trip=84，occupancy=15.46%，contact=0
Though the number of trips and occupancy decrease, tourists have more choices.

2. In the first model, the tourists travel down the river in a constant distance everyday that may make them feel tired or bored. So we set up the second model to ameliorate this problem.

3.3 Model 2

In the second model, each kind of trips provides only one route, but tourists travel different distances everyday.
Figure 3 shows a simplified outline of the second model.

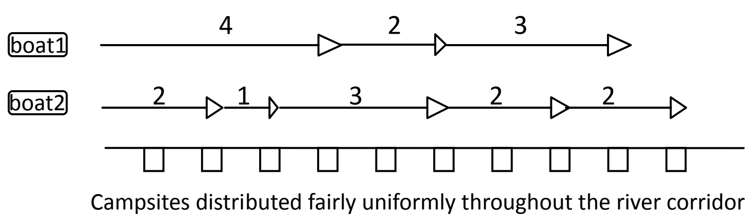


Fig.5

3.3.1 How it works

- **The process of Genetic algorithm:**

Step1. Generate the initial population $G_k = \{g_{ij}\}_k$ randomly and every initial population is the candidate solutions. Every g_{ij} is a gene which means in the population of G_k the i^{th} trip arrive at the g_{ij} campsite in the j^{th} night of the journey. Let different populations coevolved with each other. The expression of gene g_{ij}: suppose the route of the i^{th} trip is $R_i = [r_1, r_2, ..., r_i]$, r_m represents the number of the m^{th} night campsite ($6 \le m \le 18, 1 \le r_m \le Y$), so g_{ij} is a Y-dimensional vector which satisfies:

$$g_{ij} = \begin{cases} 1, if \quad j = r_m, 1 \le j \le Y, 6 \le m \le 18, \\ 0, others \end{cases}$$

Step2. The operations of the standard binary genetic algorithm are as follows:
Allow copy, mutate and crossover in a determined probability to produce the next generation in the same population.

Copy: (new) $g_i = g_i$

Mutate: arbitrarily select 1 bit from the original g_i to mutate, change the selected bit from 1 to 0 or from 0 to 1.(suppose the probability is 0.5)

Crossover: arbitrarily select n bit continuously at the beginning of g_i, g_j to exchange. (suppose the probability is 0.3)

Considering mutate and crossover:

(1) if a number in the new g_i is equal to a number in the old g_i, then get rid of g_i.

(2) if the gene is illegal: (a) the number of traveling day is more than 18 or less than 6.
(b) the traveling time of one day is more than 8 hours

then get rid of g_i.

Step3. Decode: decode from g_i to r_i ($\forall g_{ki} \in G_k$),we can get the route R_k

Step4. Evaluate R_i :

Method 1: to emulate R_i and can get the *trip, occupancy, and contact* directly.

Method 2: find an evaluation function.

Step5. Select some better solutions of g_i, if the iteration times do not meet the requirement, then turn to *step 2*, else turn to *step 6.*

Step6. Find the best solution in the rest of the populations

The genetic algorithm can find a local optimum solution, but can not guarantee that the solution is the global optimum.

3.3.2 Consequences

We suppose the number of genetic generations is 50 and Y belongs to some discrete value. Aiming at getting the largest occupancy, we can get the result in the following chart. (trip/occupancy/contact)

Y \ I	1	2	5
20	173/84.89%/0	193/86.5%/1	265/94.33%/0.2857
30	168/55.65%/0.1111	256/66.56%/0.8333	173/42.22%/0.1756
40	163/40.75%/0	330/70.82%/0.2727	191/45.56%/0.2857
50	340/62.99%/1.4286	332/54.13%/0.3	167/25.01%/0.1667

Table.4

If Y =30 and I =5, the arrangement of trips is as follows：

The date of boats setting out	Kinds of trips
1	[12,7,17,11,9,10,18,16,15,11]
6	[9,14,16,17]
11	[9,15,16,18,7]
16	[14,15,7,18,9,11,16,12,17]
21	[9,15,17,11,7]
26	[7,16]
31	[16,15]
36	[14,18,12,10,11,7,16,17,15]
41	7
46	[12,14,16,10,17]
51	[7,16,14]
56	[9,7,15,11,17,14,18,10]

61	[12,15,18,16,14,11,17,9]
66	[11,12,18,7,14,17,9,16,15]
71	[11,7,15,16,17,9,12,14]
76	[14,12,16,15,9,17,11,18,7]
81	[11,14,18,7,15,9,16,12]
86	[16,7,9,14]
91	[14,18,17,16,12]
96	[7,18,9,11,17,16]
101	[9,12,15]
106	[16,7,10]
111	[7,11,16,12,15]
116	[9,7,10,18,17,11,15,12,14]
121	[15,18,14,9,16,12,7,11,17]
126	[11,17,15,12,16,9,18,14,7]
131	[9,14,17,7,16,12,11]
136	[9,12,7,15]
141	[9,16,14,17,10]
146	15
151	[17,12,9,7,18,11,14,15]
156	[11,18]
161	[16,11,14,7,15]
166	7

Table.5

The route of each kind of trip:

T_i	The campsite that the tourists stay every night
T_{18}	2 3 4 6 13 15 16 18 19 20 21 23 24 25 26 27 28 29
T_{16}	1 5 6 8 9 10 13 14 15 17 20 22 23 24 25 26
T_{15}	5 6 9 10 13 15 18 19 20 21 22 24 27 28 29
T_{12}	1 3 6 9 13 14 16 18 19 23 24 27
T_{17}	1 3 5 12 13 14 18 19 20 21 22 23 24 25 26 27 29
T_{10}	1 5 9 11 13 14 18 21 22 26
T_{11}	1 5 8 9 10 13 14 18 20 24 28
T_9	3 4 7 10 11 14 15 16 20

T_{14}	2	5	7	9	12	14	16	17	19
	20	21	24	28	29				

Table.6

3.3.3 Sensitivity

The sensitivity analysis to Y and I:

1. To the fixed Y, we change I. Then we use the method of smooth spline, so we can get:

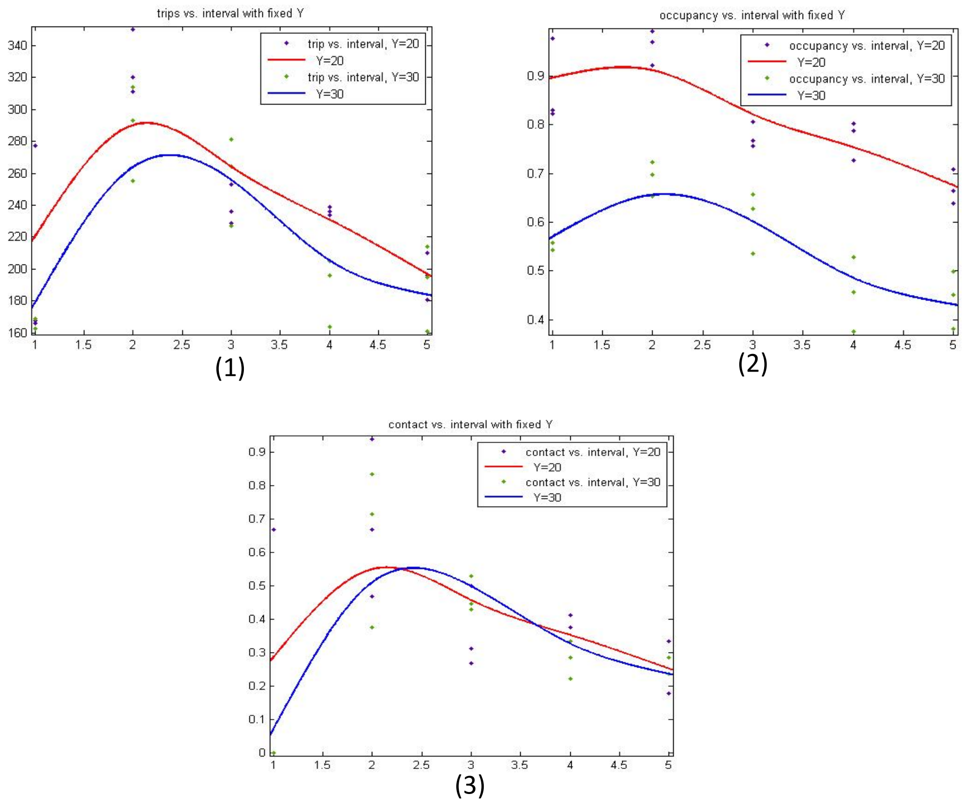


Fig. 6

From the figure, we can find that when the I $\geqslant 2$, the increase of I will lead to the decrease of the number of trip, occupancy and the number of contact.

2. To the fixed I, we change Y. Then we use the method of smooth spline, so we can get:

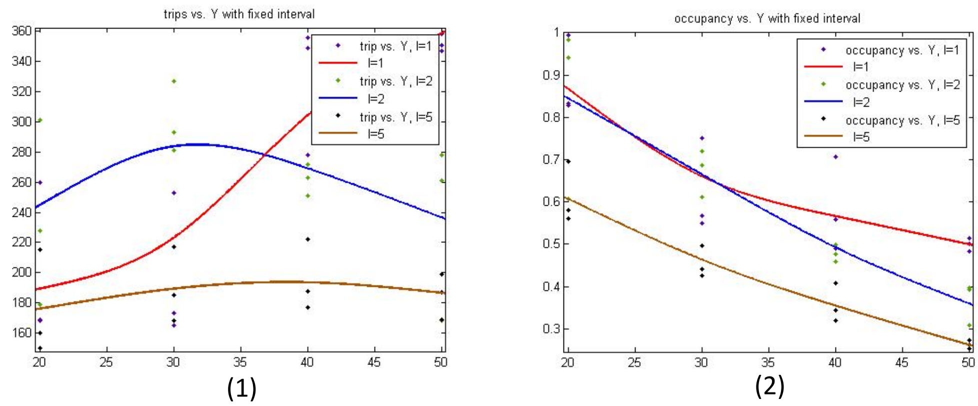


Fig.7

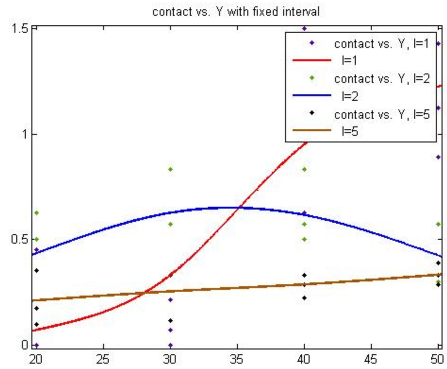


Fig.8

From the figure, we can find that Y has negative connection to occupancy and the influence of Y to the number of trip and contact decreases with the increase of I

From the analysis of Y and I, we can also find that the number of trip is in the proportion to the number of contact.

3.3.4 Weaknesses

The model use genetic algorithm to solve the problem and have some shortcomings:

We generate the initial population randomly, which may lead to a local optimum. So we can't guarantee that the result is the global optimum.

3.4 Model 3

In the third model, each kind of trips provides several routes and in each route, tourists can travel different distances everyday.

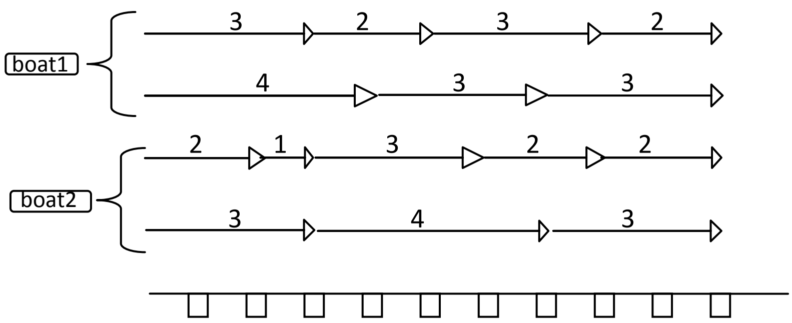


Fig.9

3.4.1 How it works

In model 2, a kind of trip only provides one route, so we need to check out whether there is more than one route of a kind of trip in the next generation. In model 3, we assume that a kind of trip provide at most *A* routes, so we will check out whether there is more than *A* routes of a trip in the next generation. If the number of the routes is more than *A*, we remain *A* newer routes and get rid of the others.

3.4.2 Consequences

We suppose the number of genetic generations is 50 and Y belongs to some discrete values. Assume that each kind of trips can at most provide three different routes. Aiming at getting the largest occupancy, we can get the result in the following chart. (trips/occupancy/contacts)

Y \ I	1	2	5
20	172/84.78%/0	290/90.11%/0.6667	210/73.64%/0.2353
30	172/56.61%/0.0667	217/61.74%/0.7143	167/44.61%/0.1111
40	174/42.44%/0	210/42.21%/0.3750	196/34.14%/0.3333
50	273/52.22%/0.0714	306/43.62%/0.7222	216/29.51%/0.3333

Table.7

3.4.3 Sensitivity

Compared with model 2, model 3 only increases the number of routes of each kind of trip, so the influence of Y and I to the optimal solution is similar to the sensitivity analysis mentioned above.

3.4.4 Weaknesses

Although the third model provides tourists more choices to select compared to the models above, it still can not meet tourists' requirement.

3.5 Model 4

All models mentioned above have stipulated routes for tourists, so tourists can have little freedom. In order to improve the quality of trips, we can establish a new kind of trip that allows tourists to decide where to go and when to have a rest. Time limited, we only have some thoughts about this model.

1. The travel agency can set the number of oar-powered rubber rafts and motorized boats everyday according to the market research.
2. After determining what kind of trip they will take part in, the tourists can decide their route freely. So collisions will occur.
3. Solution to collisions：

   Sollusion1: Every night tourists will get the information about which campsites are empty the next day, and then they can choose these campsites.

   Sollusion2 : Any trip can get the campsite if they are the first one to the campsite. Those who come later should go back or forward to other empty campsites.

   Sollusion3: Set the priority of different kind of boat.

The fourth model is a dynamic model, so the simulation of the slack season of the peak season may have a large error. But we are sure that when the number of trips increases, the occupancy will increase and the number of contacts will also increases.

4. Strengths and Weaknesses

4.1 strengths

- We do not establish a very complex model with all the variables at first. Instead, we build a simple model at first, and then increase the variable one by one. Therefore, it is easier to analyze and the influences of the variables

- The traveling hours per day and the distance of each campsite used in our mold fit the real data. In other words, our model can be successfully applied in the real world.

- We take many factors into consideration and analyze them systematically.

- We set many figures to show the influences of the variables, so it is easier for readers to understand our analyses.

- We have the sensitivity analyses of the consequences to different variables qualitatively and quantitatively.

4.2 Weakness

- Because greedy algorithm is easy to sink in to the local minimum, so the answer may not be the optimal solution.

- Genetic algorithm is easy to get local optimal solution. Besides, the calculation of genetic algorithm is huge in the models of this paper and spends much time.

- Because the genetic algorithm generate the initial population randomly, so the solution is unsteady and may be accuracy.

5. Future Work

We all use the same method to solve the problem: we aim at the highest number of trips in the premise of an acceptable number of contacts. However, different people emphasize different point, so the results will be quite different.

6. References

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